Correspondence

Characteristics of Waveguide Resonant-Iris Filters

This correspondence describes the theory and performance of a new waveguide filter that has both very wide passband and stopbands suitable for use with modern microwave generators. The filter is composed of resonant irises placed across a waveguide at quarter-wave intervals. The irises are proportioned to have the same resonant frequency but different Q-factors graded according to the maximally flat principle [1]–[7].

The theory of maximal flatness has been adapted by Mumford [1] to waveguide filters in which cavities coupled by quarter-wave-lines [8] replace the series-resonant and shunt-antiresonant branches used in lumped-constant ladder-type filters. These selective cavities are formed of obstacles (inductive posts, inductive vanes, or capacitive diaphragms) housed in waveguides [1]. In the new filter, each cavity consists of a resonant iris, making up the assembly shown in Fig. 1. As microwave components, the irises are characterized by their common resonant frequency and by their loaded-Q factors which vary in conformity with the prescribed law [1]:

$$\left.\begin{aligned} Q_1 &= Q_T \sin\frac{\pi}{2n} = Q_n \\ Q_2 &= Q_T \sin\frac{3\pi}{2n} = Q_{n-1} \\ &\cdots\cdots\cdots \\ Q_r &= Q_T \sin\frac{(2r-1)\pi}{2n} \end{aligned}\right\}. \qquad (1)$$

The factor Q_T is the total Q of the complete filter defined as

$$Q_T = 1 \bigg/ \left| \frac{f_c}{f_0} - \frac{f_0}{f_c} \right| \qquad (2)$$

where f_c is the half-power frequency. The loss function caused by the insertion of the filter between a generator and matched load is given by

$$\frac{P_0}{P_L} = 1 + \left(\frac{f}{f_0} - \frac{f_0}{f}\right)^{2n} \bigg/ \left(\frac{f_c}{f_0} - \frac{f_0}{f_c}\right)^{2n} = 1 + Q_T^{2n}\left(\frac{f}{f_0} - \frac{f_0}{f}\right)^{2n}. \qquad (3)$$

Here P_0 is the available power from the generator having an internal resistance equal to the terminal resistive load, P_L the load power with the insertion of the filter, and n the number of irises in the filter.

As shown in Fig. 2(a) and (b), the iris is simply a diaphragm containing a concentric rectangular opening. Approximately, this microwave element has the parallel LC circuit representation as in Fig. 2(c). Actually, the resonant iris constitutes a distributed-parameter component; as such, the variation of its susceptance with frequency assumes the curve [9] shown by Fig. 2(d). Because the iris, as a resonator, has a very small energy storage, it forms a very low-Q cavity. In its susceptance-versus-frequency characteristic, the first resonant frequency is widely separated from the next higher resonance. In a highly selective cavity made by inductive posts, for instance, the second resonant frequency indicated in Fig. 2(e) is not sufficiently remote from the first which causes the appearance of lowloss bands close to the passband of inductive-post filters and deterioration in their far off-band attenuation performance. In the iris filter, however, the low-Q quality of the iris leads to an extremely broad passband; the wide separation between the first and the next higher resonant frequencies results in the sustained attenuation in the near and far off-band frequency ranges. In addition, the on-band response is flat because the iris Q-factors are chosen to produce maximum flatness. The off-band attenuation can be built up to a desired level when sufficient number of irises are employed in the filter.

Based upon Slater's formula [10], the following empirical equation yields a resonant wavelength, λ_0, agreeing within ½ percent with the measured value of actual irises [11].

$$\left(\frac{a}{b}\right)\left[1 - \left(\frac{\lambda_0}{1.97a}\right)^2\right]^{1/2} = \left(\frac{D}{d}\right)\left[1 - \left(\frac{\lambda_0}{1.97D}\right)^2\right]^{1/2} \qquad (4)$$

where the symbols are defined in Fig. 2(a). Alternatively, the resonant wavelength can be calculated from the ensuing formula which is derived by using Lewin's field theory expression [12] for the normalized susceptance of tuned windows in waveguides.

$$\lambda_0 = \left[\log\csc\left(\frac{\pi d}{2b}\right)\right]^{1/2} \bigg/ \left\{ \frac{4}{\pi^2}\left(\frac{a}{b}\right)\left(\frac{D}{a^2 - D^2}\right)^2 \cot^2\left(\frac{\pi D}{2a}\right)\cos^2\left(\frac{\pi D}{2a}\right) + \frac{1}{4D^2}\log\csc\left(\frac{\pi d}{2b}\right) - \frac{1}{4abD^2}\left[\left(\frac{b^2}{3} + \frac{d^2}{2} - \frac{8bd}{\pi^2}\right) - 2\left(\frac{b}{\pi}\right)^2 \sum_{N=1}^{\infty} J_0^2\left(\frac{N\pi d}{b}\right) K\left(\frac{2N\pi(a-D)}{b}\right) \bigg/ N^2\right]\right\}^{1/2} \qquad (5)$$

where J_0 denotes the zeroth-order Bessel function and K a special function. On the basis of (4) or (5), families of curves have been calculated giving values of D as a function of the resonant frequency with d as a parameter. This makes possible the quick selection of proper sets of D and d that provide the same resonant frequency but tapered Q-factors for irises making up a filter assembly.

The resonant frequency of an already made iris can be found experimentally by measurement of the VSWR at different frequencies when the iris is inserted in a length of waveguide terminated by a matched load. Lowest VSWR varying from 1.01 to 1.04 occurs at resonance. In the vicinity of resonance, a V-curve delineates the variation of the VSWR, S, as a function of frequency. In terms of S, the normalized susceptance of the iris is expressible as [11]

$$|bn| = \frac{S-1}{\sqrt{S}}. \qquad (6)$$

The variation of bn with frequency f can be plotted to effect the determination of the slope (dbn/df) at f_0 whereby the loaded-Q of the iris can be evaluated from the relationship [11]

$$Q = \frac{f_0}{4}\left(\frac{dbn}{df}\right)_{f=f_0} = -\frac{\lambda_0}{4}\left(\frac{dbn}{d\lambda}\right)_{\lambda=\lambda_0}. \qquad (7)$$

To determine the Q-factors of irises before they are constructed, the following formula derived with the aid of (7) and Lewin's normalized susceptance expression can be employed,

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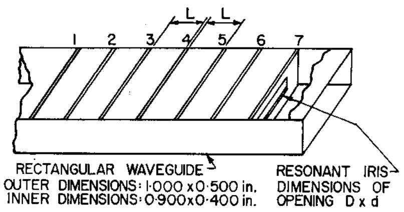

MIDBAND FREQUENCY OF FILTER MHz	RESONANT FREQUENCY OF IRIS MHz	SPACING BETWEEN TWO IRISES L (inches)	DIMENSIONS OF IRIS OPENING D x d (inches)			
			1, 7	2, 6	3, 5	4
11900	11500	0·285	0·558 x 0·108	0·542 x 0·080	0·533 x 0·060	0·528 x 0·045
11600	11200	0·300	0·569 x 0·108	0·554 x 0·080	0·546 x 0·060	0·541 x 0·045
10860	10500	0·320	0·602 x 0·108	0·589 x 0·080	0·582 x 0·060	0·577 x 0·045
10650	10300	0·330	0·611 x 0·108	0·598 x 0·080	0·591 x 0·060	0·586 x 0·045

Fig. 1. Configuration of the seven-iris bandpass filter and design data for four iris filters.

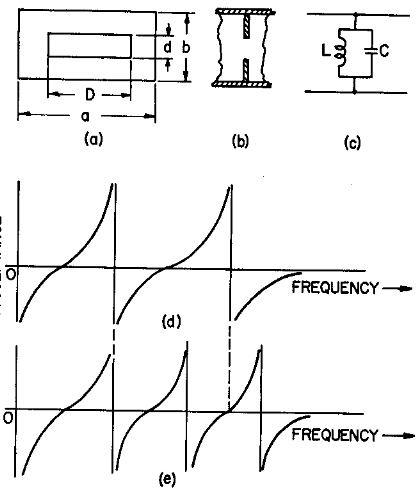

Fig. 2. (a) Dimensions of a resonant iris. (b) Disposition of the iris in waveguide. (c) Lumped-constant equivalent circuit of a resonant iris. (d) Susceptance function of a low-Q cavity made of a resonant iris. (e) Susceptance function of a cavity with high selectivity.

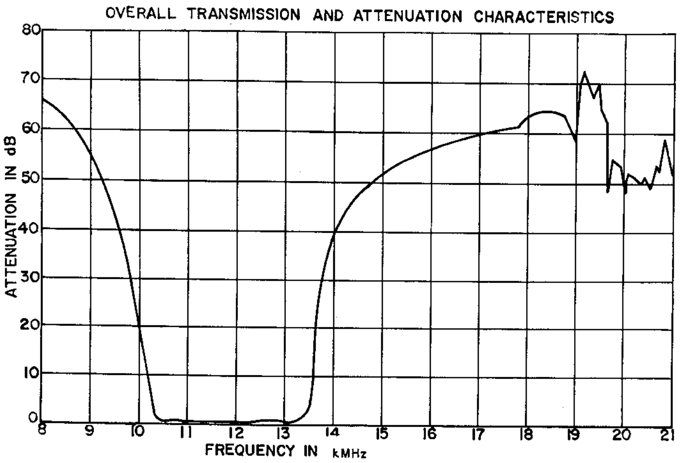

Fig. 3. Transmission and attenuation characteristics of Type 1051–11500 seven-iris filter.

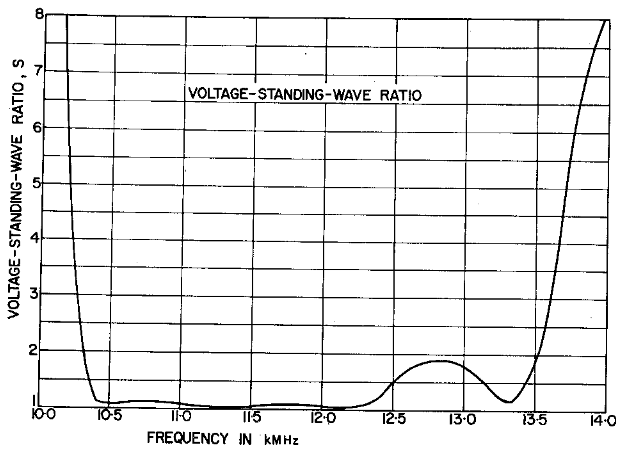

Fig. 4. Variation of voltage standing-wave ratio over the passband of Type 1051–11500 seven-iris filter.

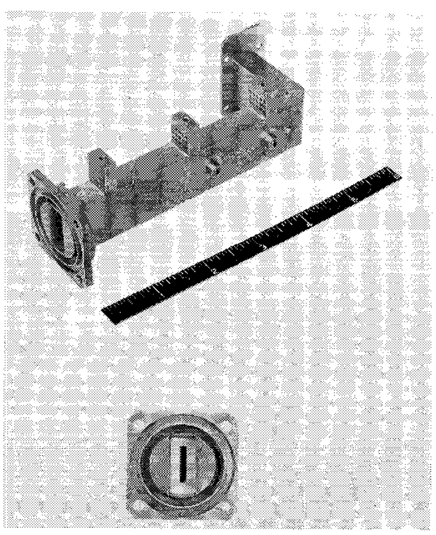

Fig. 5. (a) Perspective view of the composite filter consisting of a two-cavity inductive-post filter and a seven-iris filter. (b) End view of the composite filter. The use of the E-plane bend is optional; the bend in the filter shown here is required for a varactor frequency multiplier. The length of the long leg is $5\frac{3}{8}$ inches. The total weight is less than 3 ounces in all aluminum construction except for the two stainless steel screws used for tuning the two-cavity filter.

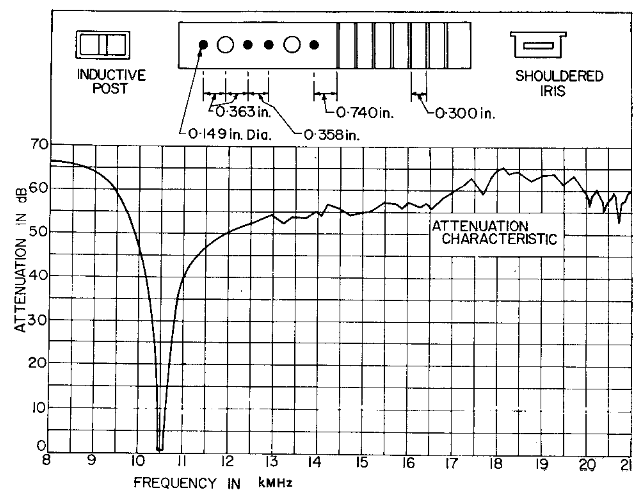

Fig. 6. Frequency response of the composite filter comprising a Type-1051 two-cavity inductive-post filter and a Type 1051-11200 seven-iris filter in tandem.

$$Q = \left(\frac{\lambda_0}{4a}\right) \frac{1}{\left(1 - \frac{\lambda_0^2}{4a^2}\right)^{3/2}} \left\{ \cot^2\left(\frac{\pi D}{2a}\right) - \frac{1}{D^2}\left[\frac{\pi(a^2 - D^2)}{4aD\cos\left(\frac{\pi D}{2a}\right)}\right]^2 \right.$$
$$\left. \times \left[\left(\frac{b^2}{3} + \frac{d^2}{2} - \frac{8bd}{\pi^2}\right) - 2\left(\frac{b}{\pi}\right)^2 \sum_{N=1}^{\infty} J_0^2\left(\frac{N\pi d}{b}\right) K\left(\frac{2N\pi(a - D)}{b}\right) \Big/ N^2\right] \right\}$$
$$+ \left(\frac{b}{\lambda_0}\right) \frac{\left(1 + \frac{\lambda_0^2}{4D^2} - \frac{\lambda_0^2}{2a^2}\right)}{\left(1 - \frac{\lambda_0^2}{4a^2}\right)^{3/2}} \left[\frac{\pi(a^2 - D^2)}{4aD\cos\left(\frac{\pi D}{2a}\right)}\right]^2 \log\csc\left(\frac{\pi d}{2b}\right). \quad (8)$$

Contours of equal Q and contours of equal resonant frequency have been mapped out in the D–d plane for irises in waveguides for different bands. These maps permit the quick determination of resonant frequency and loaded-Q for an iris of any dimension $D \times d$.

The bandwidth of the iris filter over which the VSWR is below an assigned value S is given by [11]

Iris Number	$D \times d$ inches	Resonant Frequency		Loaded-Q Measured	Total Q Q_T	Loaded Q Equation (8)
		Equation (4)	Measured			
1 7	0.558 ×0.108	11 500	11 405 11 370	1.56	6.99	1.703
2 6	0.542 ×0.080	11 500	11 420 11 500	2.11	3.36	2.233
3 5	0.533 ×0.060	11 500	11 445 11 500	2.86	3.18	2.879
4	0.528 ×0.045	11 500	11 415	3.35	3.35	3.339

$$w = f_0\left(\frac{S - 1}{2\sqrt{S}}\right)^{1/n}\left(\frac{1}{Q_T}\right) \quad (9)$$

in which f_0 is the midband frequency of the filter. The overall frequency response can be evaluated from

Loss in dB

$$= 10\log_{10}\left[1 + Q_T^{2n}\left(\frac{f}{f_0} - \frac{f_0}{f}\right)^{2n}\right]. \quad (10)$$

The on-band insertion loss decreases slowly with n, because the factor $Q_T(f/f_0 - f_0/f)$ is less than unity in the passband. The off-band attenuation rises rapidly with n, because the same factor exceeds unity in the upper and lower stopbands.

The table in Fig. 1 includes data for 4 iris filters constructed to suppress the spurious outputs in X-band varactor frequency multipliers. Because of the interaction between the closely spaced irises, the mid-frequency of the filter becomes higher than the resonant frequency of the irises composing the filter. The loaded-Q for the outer-most irises are larger than those of the others to obtain sharp cutoff at the band edges. Thus, higher Q_T than the maximally flat requirement obtains when Q_T is calculated on the basis of the loaded-Q of the outer-most irises. The attenuation close to the band edges of the filter shown in Fig. 3 agrees with the result calculated from (10) with $Q_T = 6.99$. In the far off-band ranges, the attenuation in Fig. 3 conforms to the value found from (10) but with $Q_T = 3.35$. The insertion loss amounts only to 0.2 or 0.3 dB over a major portion of the passband. The matching property of the iris filter is illustrated by Fig. 4 which indicates that the band over which S is below 2.0 extends from 10 300 to 13 500 MHz. This checks with $w = 3120$ found from (9) with $f_0 = 11\ 900$, $S = 2.0$, and $Q_T = 3.35$.

The iris filter by itself can be connected to the output of an X-band traveling-wave tube; spurious output lying outside of the passband in Fig. 3 will be eliminated. When two iris filters having overlapping passbands are cascaded, a filter of intermediate bandwidth is obtained for use in tunable solid-state generators. For narrowband applications, the iris filter is connected in tandem with a Mumford inductive-post filter, Fig. 5; the former compensates for the deficiency in attenuation found in the latter. Figure 6 portrays the frequency response of the composite filter which, when used in the output of a varactor multiplier, reduces practically all the extraneous signals to a level 80 dB below the desired signal.

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Additional Considerations in Comb-Line Bandpass Filter Interstage Couplings

In a prior correspondence,[1] various aspects of narrowband comb-line bandpass filters were discussed. Experimental data were presented showing that *nonzero* coupling is obtained between adjacent quarter-wave combline resonators when a *nonuniform* (i.e., compound) center conductor is employed. In this correspondence, additional techniques for controlling this interstage coupling will be discussed.

The comb-line filter structure previously employed has been retained herein for additional experiments with center frequencies still at 2000 MHz. From Fig. 4 of a previous correspondence,[1] a coupling bandwidth $\Delta f_{12} = 88$ MHz is obtained as the coupling with no partitions between adjacent resonators. *Nonzero* coupling exists because the compound center conductor provides *unequal* magnetic and electric couplings.

The comb-line filter structure (see Fig. 1) has been modified by use of a 0.250 inch diameter metallic coupling post located midway between the adjacent resonators and connecting the top and bottom walls of the filter. The post centerline is located 1.125 inch from the plane of the short (68.7 electrical degrees at 2000 MHz). Upon inclusion of this post into the filter structure, the measured coupling bandwidth was increased to 254 MHz.

An additional modification of the coupling mechanism entailed use of a metallic coupling screw passing through the coupling post, as shown in Fig. 1. With the end of the coupling screw flush with the exterior surface of the coupling post (i.e., gap = 1.000 inch), the mea-

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